## Advanced Management Accounting

The above equation can be restated in the logarithmic form
$\log \mathrm{Y}=\log \mathrm{a}+\mathrm{x} \log \mathrm{b}$
Cumulative Total Cost
Each of the equations (i) and (ii) defines cumulative average cost. Either of them can be converted easily to a formula for the total labour cost of all units produced up to a given point. Total cost can always be calculated from a known average cost. Hence;

Total cost $=b Y=b\left(a b^{x}\right)=a b^{x+1}$
Incremental cost
If producing a second 1000 units is to reduce cumulative average cost from sh. 10 to sh. 8 , the cost of the second 1000 units will have to be only sh. 6000 , or sh. 6 each. Hence;

| Total Cost (Sh.) | No. of Units | Average Cost (sh.) |
| :--- | :--- | :--- |
| 10,000 | 1,000 | 10 |
| $\underline{6,000}$ | $\underline{1,000}$ | $\underline{6}$ |
| $\underline{16,000}$ | $\underline{2,000}$ | $\underline{8}$ |

Defining the learning curve in terms of this incremental relationship would be more useful but is more difficult to work with. As a result, learning curve improvement ratios are usually stated as percentage reductions in cumulative average labour cost.

## Example 2:

A company makes an electronic navigational guidance system that is used for space craft, aircraft and submarines. The direct labour cost is subject to an $80 \%$ learning curve. The first unit is estimated to require 1250 direct labour hours.
Required:
Compute the average number of hours required for the first $2,3,4,8$ units.
Assume the company estimates the variable cost of producing each unit as shown;
Direct material cost sh. 40,000 per unit
Direct labour sh. 20 per hour
Variable production overhead sh. $1000+60 \%$ of direct labour cost

## Required:

Estimate the total manufacturing cost of 1,2,3,4 units of the product

## Solution:

Number of labour hours
Units

| X | Average (y) | Total $^{*}$ | Marginal cost Computations |  |
| :--- | :---: | :--- | :---: | :--- |
| 1 | 1250 | 1250 | 1250 | $\mathrm{Y}=1250 \times 1^{-0.322}=1250$ |
| 2 | 1000 | 2010 | 750 | $\mathrm{Y}=1250 \times 2^{-0.322}=1000$ |
| 3 | 878 | 2634 | 634 | $\mathrm{Y}=1250 \times 3^{-0.322}=878$ |
| 4 | 800 | 3200 | 566 | $\mathrm{Y}=1250 \times 4^{-0.322}=800$ |
| 8 | 640 | 5120 | $\mathrm{Y}=1250 \times 8^{-0.322}=640$ |  |

*Total hours = Average labour hours x no. of units

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Re-order Levels $S$ | Buffer Stock B | Expected Stock-out per order | Expected Annual Shortage | Expected stock-out cost (£) | Holding Cost (£) | Total cost <br> (£) |
| 4.69 | 0 | 0.5974 | 4.978 | 19.912 | 0 | 19.912 |
| 5 | 0.31 | 0.43 | 3.58 | 14.32 | 1.55 | 15.87 |
| 6 | 1.31 | 0.19 | 1.583 | 6.332 | 6.55 | $12.882 \underset{$ minimum  <br>  cost $}{\leftarrow}$ |
| 7 | 2.31 | 0.05 | 0.4166 | 1.664 | 11.55 | 13.214 |
| 8 | 3.31 | 0 | 0 | 0 | 16.55 | 16.55 |
|  |  |  | Working <br> column 3 x <br> $\frac{250}{30}$ | $\begin{aligned} & \text { Column } 4 \mathrm{x} \\ & 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Column 2 } \\ & \text { x } 5 \text { i.e. } \\ & \text { B x h } \end{aligned}$ |  |

Hence for minimum cost $S=6$

Buffer stock
and minimum cost

$$
S=6-4.69
$$

$$
=1.31 \text { tones }
$$

$$
=£ 12.88
$$

## Workings and explanations

1. When Re-order Level

$$
\begin{array}{ll}
\mathrm{S} & = \\
\mathrm{B} & =0.69 \\
& 0
\end{array}
$$

Hence possible shortages will occur when demand is $5,6,7$ or 8 units.

| Demand | Shortage $x$ | Prob $\mathrm{p}(x)$ | Expected Value $x \mathrm{p}(x)$ |
| :--- | :--- | :--- | :--- |
| 5 | $5-4.69$ | 0.30 | $0.31 \times 0.30=0.93$ |
| 6 | $6-4.69$ | 0.10 | $1.31 \times 0.10=0.131$ |
| 7 | $7-4.69$ | 0.09 | $2.31 \times 0.09=0.2079$ |
| 8 | $8-4.69$ | 0.05 | $3.31 \times 0.05=0.1655$ |
|  |  | $\sum \times \mathrm{p}(x)$ | 0.5974 |

This is expected stock out per order when demand is $5,6,7$ or 8
2. Similarly if Re-order level is 5 Shortages
will occur if demand is $6,7,8$
Working in the way as above

| Demand | Shortage $x$ | Prob p(x) | Expected Stock-out <br> cost |
| :--- | :--- | :--- | :--- |
| 6 | 1 | 0.10 | 0.10 |
| 7 | 2 | 0.09 | 0.18 |
| 8 | 3 | 0.05 | 0.15 |
|  |  | $\sum x p(x)$ | 0.43 |

## QUESTION TWO

|  | Y |  |  | Row minimum |
| :---: | :---: | :---: | :---: | :---: |
| X | 1 | 2 | -1 | -1 |
|  | -2 | 1 | 1 | -2 |
|  | 2 | 0 | 1 | 0 |
| Column <br> maximum | 2 | 2 | 1 |  |

There is no saddle point
Let the three probabilities of Y be $\mathrm{p}, \mathrm{q}, \mathrm{r}$
The three payoffs to Y corresponding to each of the three moves of his opponent X must all be equal to the optimal value of $V$ of the game.

Y's payoffs against the three moves of X are

$$
1 \mathrm{p}+2 \mathrm{q}+(-1 \mathrm{r})-2 \mathrm{p}+1 \mathrm{q}+1 \mathrm{r} \quad 2 \mathrm{p}+0 \mathrm{q}+1 \mathrm{r}
$$

We obtain three equations by equating each of these payoffs to V

$$
1 p+2 q-1 r \quad=\quad-2 p+1 q+1 r=2 p+o q+1 r=V
$$

Also $\mathrm{p}+\mathrm{q}+\mathrm{r}=1 \quad$ (Total probability)
$1 \mathrm{p}+2 \mathrm{q}-1 \mathrm{r} \quad=\quad-2 \mathrm{p}+1 \mathrm{q}+1 \mathrm{C}$
$1 \mathrm{p}+2 \mathrm{q}-1 \mathrm{r} \quad=\quad 2 \mathrm{p}+\mathrm{oq}+1 \mathrm{r}[2]$
$\mathrm{p}+\mathrm{q}+\mathrm{r} \quad=1 \square$ [3]
Solving these three equations simultaneously we get

$$
\mathrm{p}=\frac{2}{17} \quad \mathrm{q}=\frac{8}{17} \text { and } \mathrm{r}=\frac{7}{17}
$$

Similarly using the same reasoning as before, let the three probabilities of x be $\mathrm{p}^{\prime}, \mathrm{q}^{\prime}, \mathrm{r}^{\prime}$,
We get $1 \mathrm{p}^{\prime} 2 \mathrm{q}^{\prime}+2 \mathrm{r}^{\prime}=2 \mathrm{p}^{\prime}+1 \mathrm{q}^{\prime}+$ or' $=-1 \mathrm{p}^{\prime}+1 \mathrm{q}^{\prime}+1 \mathrm{r}^{\prime}$
Also

$$
\mathrm{p}^{\prime}+\mathrm{q}^{\prime}+\mathrm{r}^{\prime}=1
$$

Solving them simultaneously we get

$$
\begin{array}{lrr}
\mathrm{p}^{\prime}=3 & \mathrm{q}^{\prime}=5 \mathrm{r}^{\prime}= & \underline{9} \\
17 & 17
\end{array}
$$

Hence X should play his rows in the ratio 3:5:9 (randomly)
Y should play his columns in the ratio 2:8:7

The reported financial statements would also be as follows:

|  | Division | Division |
| :--- | :---: | :--- |
|  | $\mathbf{A}$ | $\mathbf{B}$ |
| Sales revenue (external customers) | $\mathbf{£}^{\prime} \mathbf{0 0 0}$ | $\mathbf{£}^{\prime} \mathbf{0 0 0}$ |
| Transfers (at $£ 22.20$ per unit) | 3,000 | 1,250 |
|  | $\underline{5555}$ | $\underline{(555)}$ |
| Variable cost (excluding transfers) | $\underline{1,555}$ | 695 |
| Contribution | $\underline{1,680}$ | $\underline{250}$ |
| Fixed cost | $\underline{500}$ | $\underline{225}$ |
| Profit | $\underline{\underline{6,625}}$ | $\underline{\underline{220}}$ |
| Investment | $\underline{17.8 \%}$ | $\underline{17.6 \%}$ |
| Return on investment |  |  |

There are two aspects to the behavioural aspects of this situation which will be discussed. The first concerns the extent to which managers of A and B would find the transfer price 'fair'. Any attempt by the management accountant to impose a transfer price would be perceived to be an infringement of autonomy and may lead to dysfunctional consequences. Wherever possible, if the autonomy of the division is to be guarded and an imperfect market operates, negotiated prices appear to offer most prospects of optimising the behavioural implications. The second behaviouralimplication concerns the motivation of managers to accept worthwhile projects. If it is accepted that managers are motivated to improve their reported performance, performance measures which lead to managers rejecting profitable projects are dysfunctional. This particular idea can be explored in relation to

## Example 1.

At the existing transfer price of $£ 30$, the manager of B would produce the following calculations of the value of opening the branch office:

Additional sales 5,000 @£50 £250,000 Additional variable costs:
Transfer price
Other variable costs £ 50,000
Fixed costs $£ 50,000 \quad £ 250,000$
Net profit


On behavioural grounds, the project would be rejected by the manager because performance does not improve as a result of the effort necessary to open the branch. However, from Conglom plc's point of view the calculation would appear as follows:

| Additional sales 5,000 @£50 |  | $£ 250,000$ |
| :--- | :--- | ---: |
| Additional variable costs: |  |  |
| Transfer price | $£$ nil |  |
| Other variable costs | $£ 125,000$ |  |
| Fixed costs | $\underline{£ 50,000}$ | $\underline{£ 175,000}$ |
| Net profit |  | $£$$£ 75,000$ |

It is advantageous to the company as a whole, for the branch office to be opened. Since A has spare capacity sufficient to meet the additional requirement, a transfer price equal to the variable costs incurred in division A would lead to the manager of department B making the correct decision. A transfer price between $£ 15$ and $£ 30$ would lead to the branch being opened but a transfer price of $£ 15$ alone would ensure that all future decisions were evaluated correctly at divisional level. This leads to the second point in the critique performance measurement.

